Chapter 3 Beam Optics

- An important paraxial wave solution that satisfies Helmholtz equation is Gaussian beam. Example: Laser.

3.1 The Gaussian Beam

A. Complex Amplitude

$$U(\vec{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \qquad \text{Amplitude factor}$$

$$\times \exp\left\{-j\left[kz - \tan^{-1}\left(\frac{z}{z_0}\right)\right]\right\} \qquad \text{Longitudinal phase} \qquad (3.1\text{-}7)$$

$$\times \exp\left[-j\frac{k\rho^2}{2R(z)}\right] \qquad \text{Radial phase}$$

$$W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} \qquad (3.1\text{-}8)$$

$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right] \qquad (3.1\text{-}9)$$

$$z_0 \equiv \frac{\pi W_0^2}{\lambda} \qquad (3.1\text{-}11)$$

→ Knowing W_0 and λ (or z_0), a Gaussian beam is determined!

Ref: Verdeyen, "Laser Electronics," Chapter 3, Prentice-Hall

B. Properties of Gaussian Beam

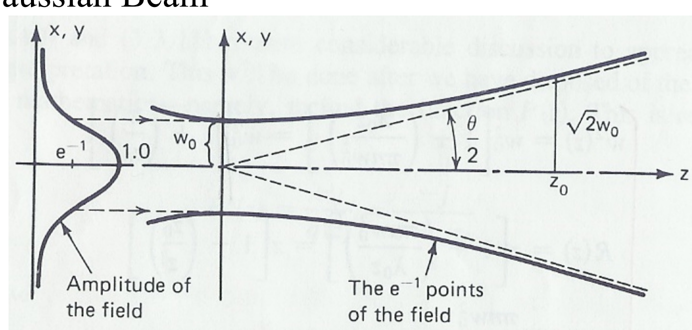

Figure 3.2 Spreading of a $TEM_{0,0}$ mode.

<u>Intensity and power</u>

$$I(\rho,z)=I_0\left[\frac{W_0}{W(z)}\right]^2\exp\left[-\frac{2\rho^2}{W^2(z)}\right] \tag{3.1-12}$$

$$P=\frac{1}{2}I_0\left(\pi W_0^2\right) \tag{3.1-14}$$

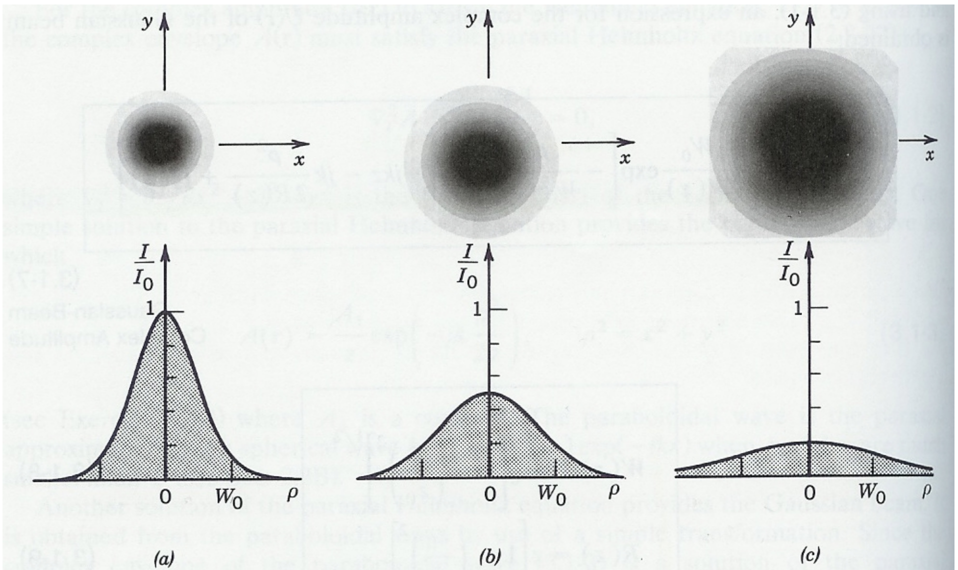

Figure 3.1-1 The normalized beam intensity I/I_0 as a function of the radial distance ρ at different axial distances: (*a*) $z=0$; (*b*) $z=z_0$; (*c*) $z=2z_0$.

<u>Beam radius and divergence</u>

$$I(\rho,z)=\frac{1}{e^2}I(0,z)\ \text{ when } \rho=W(z)$$

$$W(z)=W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}\ \text{: Beam radius} \tag{3.1-17}$$

W_0: Waist radius

$2W_0$: Spot size

$$W(z_0)=\sqrt{2}W_0$$

$$\theta_0=\frac{\lambda}{\pi W_0}\text{: Divergence angle (}=\frac{\theta}{2}\text{ in Fig. 3.2 shown above)} \tag{3.1-19}$$

→ Highly directional beam requires short λ and large W_0.

<u>Depth of focus (Confocal parameter)</u>

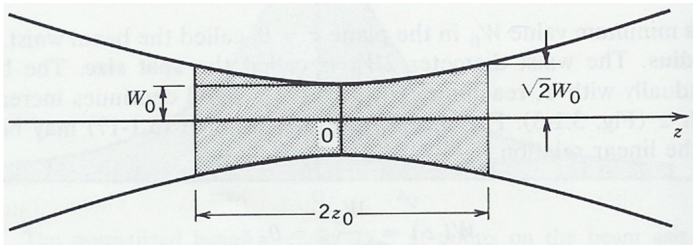

Figure 3.1-4 The depth of focus of a Gaussian beam.

$$2z_0 = \frac{2\pi W_0^2}{\lambda} \qquad (3.1\text{-}21)$$

<u>Longitudinal phase</u>

$$\varphi(z) = kz - \tan^{-1}\left(\frac{z}{z_0}\right)$$

→ Second term: phase retardation

Phase velocity $\quad v_p = \left(\frac{\varphi}{\omega z}\right)^{-1} = \dfrac{c}{1 - \dfrac{\lambda}{2\pi z}\tan^{-1}\left(\dfrac{z}{z_0}\right)} > c\,!$

<u>Wavefront bending</u>

Surface of constant phase velocity:

$$z + \frac{\rho^2}{2R} = q\lambda + \xi\frac{\lambda}{2\pi}, \quad \xi \equiv \tan^{-1}\left(\frac{z}{z_0}\right)$$

→ Parabolic surface with radius of curvature R

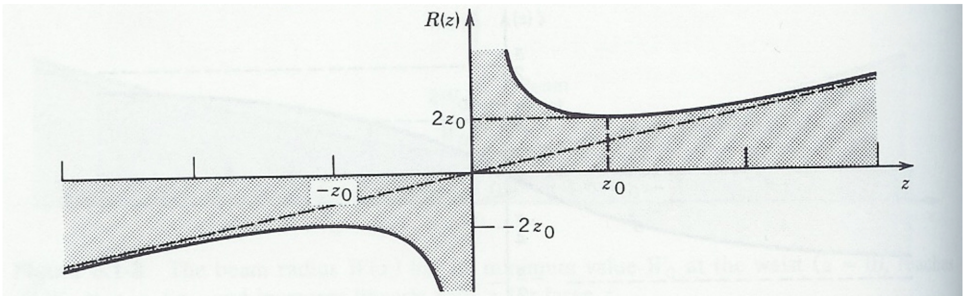

Figure 3.1-6 The radius of curvature $R(z)$ of the wavefronts of a Gaussian beam. The dashed line is the radius of curvature of a spherical wave.

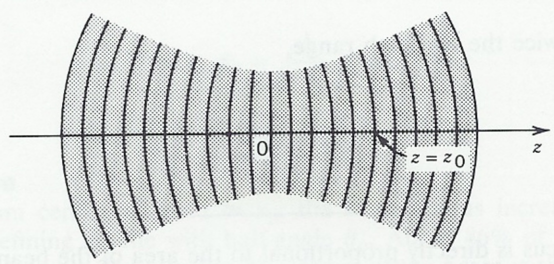

Figure 3.1-7 Wavefronts of a Gaussian beam.

3.2 Transmission through Optical Components

Gaussian beam remains a Gaussian beam after transmitting through a set of circularly symmetrical optical components aligned with the beam axis. Only the beam waist and curvature are altered.

A. Transmission through a Thin Lens

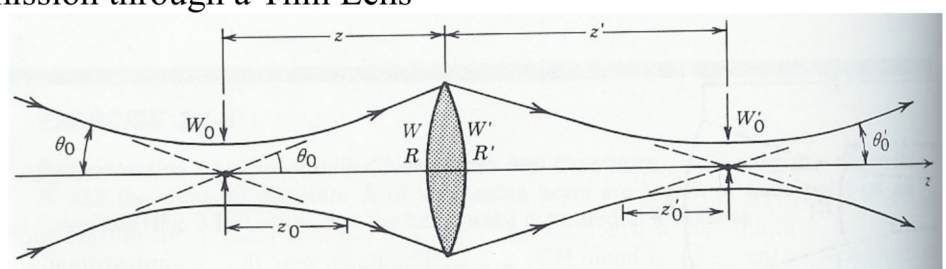

Figure 3.2-1 Transmission of a Gaussian beam through a thin lens.

$$\frac{1}{R'} = \frac{1}{R} - \frac{1}{R'} \qquad (3.2\text{-}2)$$

$$W' = W$$

$$W_0' = \frac{W}{\sqrt{1 + \left(\frac{\pi W^2}{\lambda R'}\right)^2}} \qquad (3.2\text{-}3)$$

$$z' = \frac{R'}{1 + \left(\frac{\lambda R'}{\pi W^2}\right)^2} \qquad (3.2\text{-}4)$$

Waist radius $\quad W_0' = MW_0 \qquad$ (3.2-5)

Waist locatioin $\quad (z' - f) = M^2 (z - f) \qquad$ (3.2-6)

Depth of focus $\quad 2z_0' = M^2 (2z_0) \qquad$ (3.2-7)

Divergence $\quad 2\theta_0' = \frac{2\theta_0}{M} \qquad$ (3.2-8)

Magnification $\quad M = \frac{M_r}{\sqrt{1 + r^2}} \qquad$ (3.2-9)

$$M_r \equiv \left|\frac{f}{z - f}\right|, \quad r \equiv \frac{z_0}{z - f} \qquad (3.2\text{-}9a)$$

Example: A planar wave transmitting through a thin lens is focused at distance $z' = f$.

B. Beam Shaping

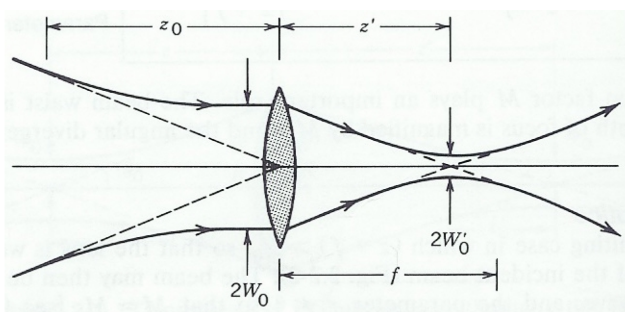

Figure 3.2-3 Focusing a beam with a lens at the beam waist.

Waist of incident Gaussian beam is at lens location.

$$R' = -f$$

$$W_0' = \frac{W_0}{\sqrt{1 + \left(\frac{z_0}{f}\right)^2}} \qquad (3.2\text{-}13)$$

→ To focus into a small spot, we need large incident beam width, short focal length, short wavelength.

$$z' = \frac{f}{1 + \left(\frac{f}{z_0}\right)^2} \tag{3.2-14}$$

F number of a lens

$$F_\# \equiv \frac{f}{D}$$

$D = 2W_0$: Diameter of the lens

Focal spot size $$2W_0' = \frac{4}{\pi}\lambda F_\# \tag{3.2-17}$$

C. Reflection from a Spherical Mirror

Same as transmission through a thin lens, $f = -R/2$

$$W_2 = W_1 \tag{3.2-19}$$

$$\frac{1}{R_2} = \frac{1}{R_1} + \frac{2}{R} \tag{3.2-20}$$

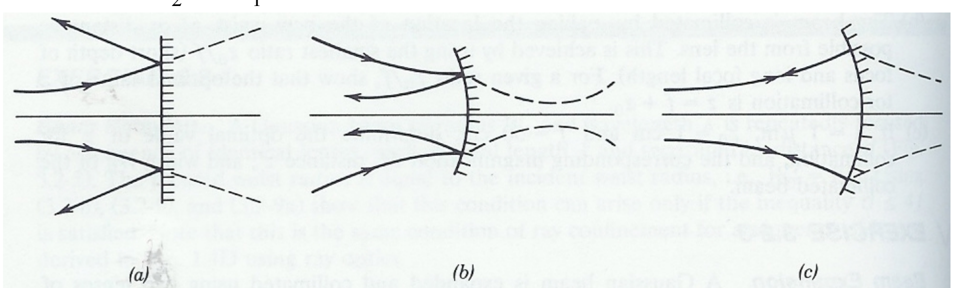

Figure 3.2-8 Reflection of a Gaussian beam of curvature R_1 from a mirror of curvature R: (*a*) $R = \infty$; (*b*) $R_1 = \infty$; (*c*) $R_1 = -R$. The dashed curves show the effects of replacing the mirror by a lens of focal length $f = -R/2$.

D. Transmission through an Arbitrary Optical System

The ABCD Law

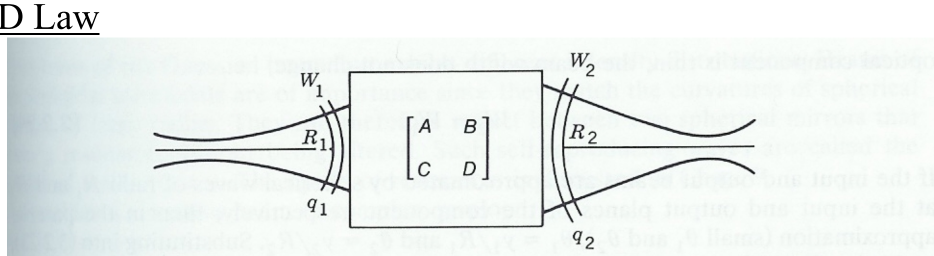

Figure 3.2-9 Modification of a Gaussian beam by an arbitrary paraxial system described by an *ABCD* matrix.

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \tag{3.2-9}$$

Like in the case of ray-transfer matrix, the ABCD matrix of a cascade of optical components (or systems) is a product of the ABCD matrices of the individual components (or systems).

3.3 Hermite-Gaussian Beams

Modulated version of Gaussian beam

→ Intensity distribution not Gaussian, but same wavefronts and angular divergence as the Gaussian beam.

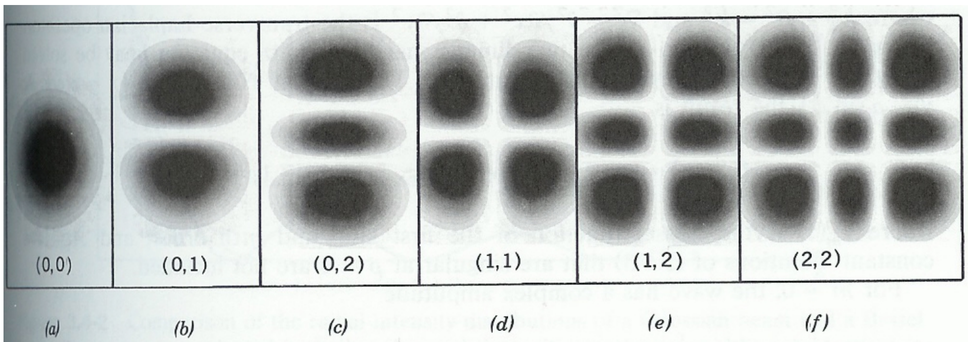

Figure 3.3-2 Intensity distributions of several low-order Hermite–Gaussian beams in the transverse plane. The order (l, m) is indicated in each case.